647510 **BSCalgCC5010** **Seat No:** __________

## B.Sc. Semester - 5 (CBCS) Examination
## Oct/Nov. - 2019
## MATHEMATICAL ANALYSIS-1 AND ABSTRACT ALGEBRA-1 (CORE)

**Time: 2:30 Hours** **Marks: 70**

**Instructions:**

1. All questions are compulsory.
2. Figures to the right indicate marks.

---

**Q-1**

**(A) ATTEMPT ANY TWO:** **10**

**(1)** State and prove the *Housedorff's* principle for metric space.

**(2)** Prove that the closure of a subset in a metric space is a closed set.

**(3)** Prove that $(R, d)$ is a separable metric space where $d$ is the usual metric.

**(B) ATTEMPT ANY ONE :** **04**

**(1)** If $(X, d)$ is a metric space, then prove that $\left(X, \frac{d}{1+d}\right)$ is also a metric space.

**(2)** Is $\frac{101}{102} \in C$ ? Justify. (where $C$ is the Cantor Set)

**Q-2**

**(A) ATTEMPT ANY TWO:** **10**

**(1)** State and prove Darboux's theorem.

**(2)** State and prove the necessary and sufficient conditions for a function to be R-integrable.

**(3)** Evaluate $\int_0^1 f(x)dx$ by showing that $f$ is R-integrable in [0,1] where f is defined as follows:

$$f(x) = \frac{1}{a^r - 1}; if \frac{1}{a^r} < x \leq \frac{1}{a^{r-1}}, r = 1,2,3 \ldots$$

$$= \quad 0; if x = 0$$

where $a > 1$.

**(B) ATTEMPT ANY ONE :** **04**

**(1)** $f(x) = \frac{1}{x+1}, x \in [0,1], P = \left\{0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right\}$.Find U(P,f) and L(P,f).

**(2)** Evaluate $\int_0^1 e^x dx$ by using the definition of $R$-integration.

**Q-3**

**(A) ATTEMPT ANY TWO:** **10**

**(1)** Let $f, g \in R_{[a,b]}$ be such that $f$ is continuous and $g$ is invariant in sign on $[a, b]$.
Then, prove that for some $c \in [a, b]$, $\int_a^b f(t)g(t)dt = f(c) \int_a^b g(t)dt$.

**(2)** Define: Group
Prove that the set of all $n^{th}$ roots of unity is a commutative group under the usual multiplication.

**(3)** Let G be a group.
Then, prove that, (i) $O(a) = O(a^{-1}), \forall a \in G$
(ii) $O(bab^{-1}) = O(a), \forall a, b \in G$.

**(B) ATTEMPT ANY ONE :** **04**

**(1)** Evaluate: $\lim_{n\to\infty} \left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\ldots\left(1+\frac{4n}{n}\right)\right]^{\frac{1}{n}}$

**(2)** Prove that
$G = \{f_{ab}/f_{ab} : R \to R, f_{ab}(x) = ax + b, \forall x \in R for given\ a, b \in R, a \neq 0\}$is a group w.r.t. the binary operation of composition of mappings.

**Q-4**

**(A) ATTEMPT ANY TWO:** **10**

**(1)** Prove that a finite non-empty subset H of a group G is a subgroup if H is closed under the binary operation of G.

**(2)** Let G be a group and $a \in G$. Then, prove that $H = \{a^n/n \in Z\}$ is the smallest cyclic subgroup containing $a$.

**(3)** State and prove the Lagrange's Theorem.

**(B) ATTEMPT ANY ONE :** **04**

**(1)** State and prove the Fermat's Theorem.

**(2)** Find the remainder of $7^{1000}$ when divided by 24.

**Q-5**

**(A) ATTEMPT ANY TWO:** **10**

**(1)** Prove that a subgroup H of a group G is a normal subgroup ***iff*** the product of any two right cosets of H in G is again a right coset of H in G.

**(2)** Prove that the relation ' $\cong$ ' of isomorphism of groups is an equivalence relation.

**(3)** State and prove the Cayley's Theorem.

**(B) ATTEMPT ANY ONE:** **04**

**(1)** If H and K are normal subgroups of a group G such that $H \cap K = \{e\}$, then prove that $hk = kh, \forall h \in H, \forall k \in K$.

**(2)** Prove that an infinite cyclic group is isomorphic to the group of integers.

*******************